Polyomino Tilings

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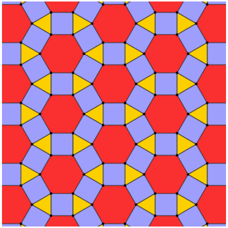
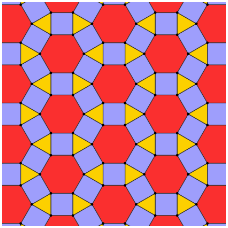
O posadzkach

Tiling

1. Tiling $\mathcal{T}$, a covering of the plane with non-overlapping shapes.

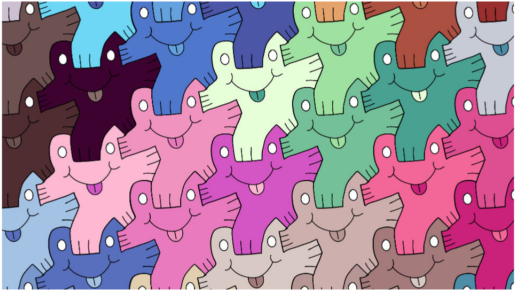


1. A symmetry of a tiling is any plane isometry that can be used to map all the tiles to other tiles in an exact fashion. I.e. a tile is mapped to a (possibly different) tile.
2. E.g. rotations, reflections, translations.

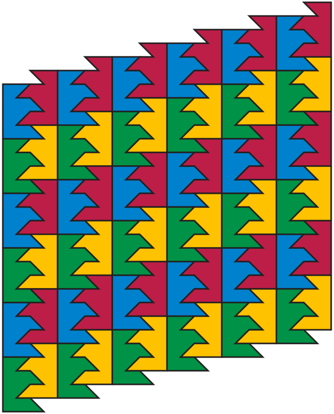


Isohedral tiling

1. Isohedral tiling is a tiling where any two tiles $T_1, T_2 \in \mathcal{T}$ can be mapped to each other using a symmetry of the tiling.

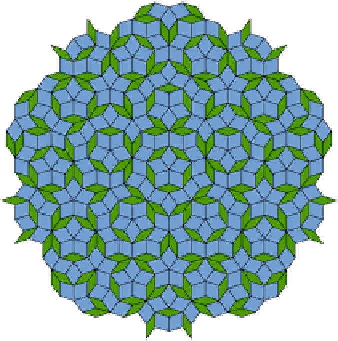


1. Non-isohedral tiling is where there are two tiles $T_1, T_2 \in \mathcal{T}$ that cannot be mapped onto each other using a symmetry of the tiling.

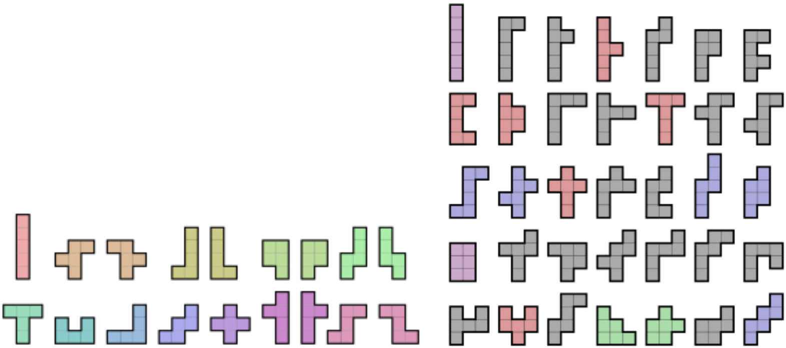


Aperiodic tiling

1. Aperiodic tilings are tilings with no translational symmetry.
2. Penrose tiling with two prototiles.
3. Is there an aperiodic tiling with one prototile? OPEN



Polyominos



Polyomino tiling problem

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2. It is decidable and there is a known optimal algorithm in $O(n)$, n is the number of edges of the polyomino.

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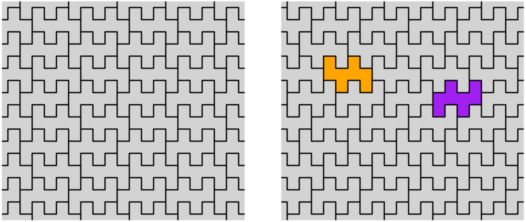

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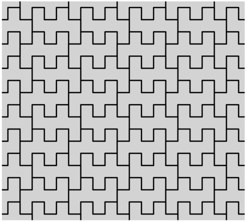

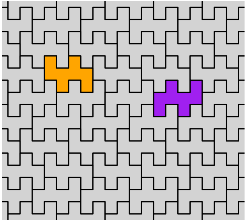

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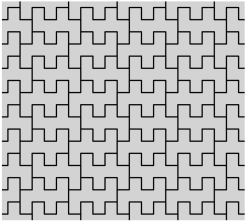

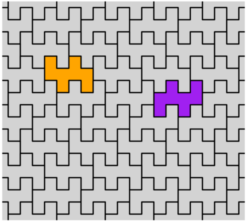

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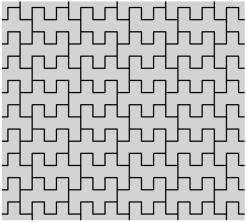
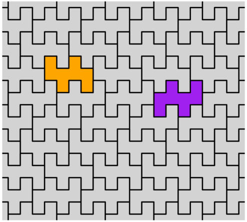
4. Open problem, does there exist an $O(n)$ algorithm for determining whether a polyomino admits a isohedral tiling?





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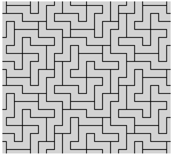

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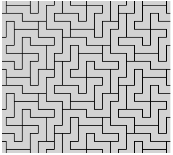

3. Does every polyomino with a tiling consisting of only translated and rotated by 180° copies of the prototile also has such a tiling that is isohedral. OPEN

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Aperiodic polyomino tilings

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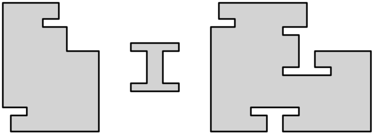

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3. Is there an algorithm that answers the question but for sets of up to 4 polyomino's? OPEN

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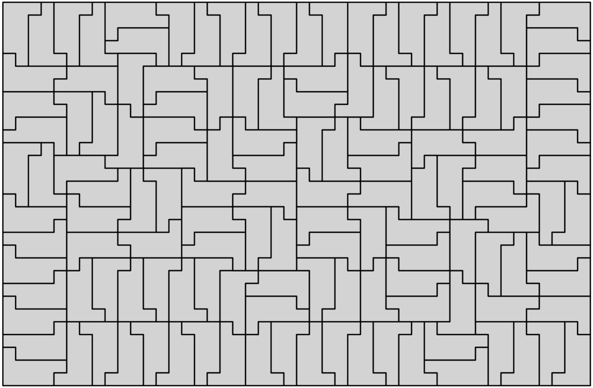
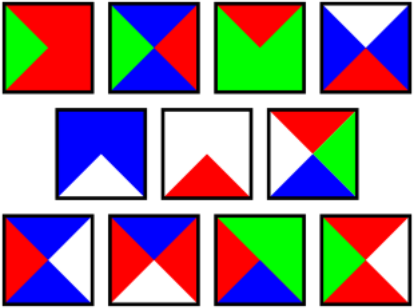
7. Does there exist a polyomino of order 5? OPEN

Polyomino of order 138



Undecidable tiling



Dziękuję za uwagę!